Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Winter Examination-2018

Subject Name: Real Analysis

Subject Code: 5SC02REA1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 23/10/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 Answer the Following questions: **(07)**

a) Define: Algebra of sets **(02)**

b) If $A, B \subseteq R$ be such that $m^*(B)=0$ then prove that $m^*(A\cup B)=m^*(A)$. **(02)**

c) If $A=\{1,2,3,....,10\}$ then find $m^*(A)$. **(01)**

d) True or false: Every union of algebra is an algebra of X. **(01)**

e) Define: F_σ - set **(01)**

Q-2 Attempt all questions **(14)**

a) Let $\mathcal{A}$ be an algebra on X and $\{A_i\}\in\mathcal{A}$ then there exist $\{B_i\}\in\mathcal{A}$ such that **(06)**

i) $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$ and ii) $B_i \cap B_j = \phi$, for $i \neq j$.

b) Prove that outer measure of an interval is its length. **(08)**

OR

Q-2 Attempt all questions **(14)**

a) Prove that P is non-measurable set. Where P contains one element from each equivalence classes E_λ and $\bigcup E_\lambda = X = [0,1)$. **(10)**

b) Consider $X = R$ and $\mathcal{A}=\{A\in R/\text{either A or A}^c \text{ is countable}\}$ then show that $\mathcal{A}$ is a σ-Algebra on R. **(04)**

Q-3 Attempt all questions **(14)**

a) Prove that every borel set is a measurable set. **(05)**

b) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*\left(A \cap E_i\right)$. **(05)**

c) Give an example of an algebra which is not an σ-Algebra of X and explain. **(04)**

OR

Q-3 Attempt all questions **(14)**

a) Give an example of measurable map which is not a Riemann integrable map and explain. **(05)**

b) Let ϕ and ψ are simple functions on E which are vanish outside of a set of finite measure then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$. **(05)**

c) Let E_1 and E_2 be two measurable subsets of R then prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$. **(04)**

SECTION – II

Q-4 Answer the Following questions: **(07)**

a) State Fatou's lemma. **(02)**

b) Define: $BV[a,b]$. **(02)**

c) State Egoroff's theorem. **(02)**

d) Define: Lebesgue integral. **(01)**

Q-5 Attempt all questions **(14)**

a) State and prove Bounded convergence theorem. **(07)**

b) State and prove monotone convergence theorem. **(07)**

OR

Q-5 Attempt all questions **(14)**

a) Let f be a bounded measurable function on $[a,b]$ and $F(x) = \int_a^x f(t)dt + F(a)$ then $F'(x) = f(x)$ almost everywhere on $[a,b]$. **(07)**

b) State and prove Lebesgue dominated convergence theorem. **(07)**

Q-6 Attempt all questions **(14)**

a) F is absolutely continuous function on $[a,b]$ iff F is indefinite integral. **(07)**

b) State and prove Beppo-Levi's theorem. **(05)**

c) Write Chebychev's inequality. **(02)**

OR

Q-6 Attempt all Questions **(14)**

a) Suppose $f, g \in BV[a,b]$ then prove the following: **(07)**

i) $fg \in BV[a,b]$ and ii) $\frac{f}{g} \in BV[a,b], where\ g \neq 0$.

b) If f is integrable over E then $|f|$ is integrable over E and $\left|\int_E f\right| \leq \int_E |f|$. **(04)**

c) If $A, B \subseteq R$ be such that $m^*(A) = 0$ then prove that $m^*(A \cup B) = m^*(B)$. **(03)**